

# On Non-Critical Superstring/Black Hole Transition

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## Abstract

An interesting case of string/black hole transition occurs in two-dimensional non-critical string theory dressed with a compact CFT. In these models the high energy densities of states of perturbative strings and black holes have the same leading behavior when the Hawking temperature of the black hole is equal to the Hagedorn temperature of perturbative strings. We compare the first subleading terms in the black hole and closed string entropies in this setting and argue that the entropy interpolates between these expressions as the energy is varied. We compute the subleading correction to the black hole entropy for a specific simple model.

## 1. Introduction and Summary

The microscopic origin of black hole entropy is a fundamental question to which string theory provides many clues. For technical reasons, it is often easier to make precise statements about BPS black holes at zero temperature. Their finite-temperature cousins, although harder to study, are nonetheless interesting in their own right. Consider the Schwarzschild black hole in four dimensions, whose Schwarzschild radius is proportional to the mass  $r \sim M l_p^2$  (where  $l_p$  is the Planck length). When the mass (energy) is very large, the string corrections to classical gravity are negligible, the entropy scales like  $S \sim M^2 l_p^2$  and hence black holes dominate the spectrum. Following [1,2], let us consider the value of mass for which the curvature near the horizon becomes order one in string units,  $M \sim l_s/l_p^2 \sim 1/g_s^2 l_s$ . The black hole entropy at this point,  $S \sim M l_s$ , is equal to that of perturbative string, up to a numerical factor.

Recently D. Kutasov [3] proposed a way to determine the correspondence point precisely. Suppose we know the Schwarzschild solution within the string theory. Knowing the solution to all orders in  $\alpha'$  is important precisely because in the vicinity of the correspondence point  $\alpha'$  corrections to the classical gravity become large near the horizon. The euclidean solution is asymptotically flat and the circumference of the temporal direction asymptotes to the inverse Hawking temperature  $\beta$ . According to [3], whenever  $\beta$  is equal to the Hagedorn temperature of perturbative strings, the entropies of the black hole and fundamental strings agree.

To test these ideas [3], one can look at the two-dimensional black hole, which is described in string theory by an exactly solvable CFT. The cleanest example is  $\mathcal{N} = 2$  supersymmetric  $SL(2)_k/U(1)$  with central charge  $c = 3 + 6/k$ , which should be dressed with an additional matter CFT to make string theory critical. The black hole has a Hagedorn density of states (to be reviewed below) with the Hagedorn temperature equal to the Hawking temperature,

$$\frac{1}{T} = \beta = 2\pi\sqrt{k\alpha'} \quad (1.1)$$

This should be compared with the Hagedorn temperature of perturbative strings

$$\beta_H = 2\pi\sqrt{(2 - 1/k)\alpha'} \quad (1.2)$$

Hence, for  $k > 1$  the black holes dominate the spectrum, while at the correspondence point  $k = 1$  determined by  $\beta = \beta_H$ , the entropies of black holes and strings are, of course, the same [3]<sup>1</sup>. For  $k < 1$  black hole drops out of the physical spectrum [5].

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<sup>1</sup> Similar picture holds for charged two-dimensional black holes [4].

In this paper we are mostly concerned with the theory at the correspondence point ( $k = 1$ ). Consider the background

$$\mathbb{R}_t \times \mathbb{R}_\phi \times \mathbf{S}^1 \times \mathbf{X} \quad (1.3)$$

where  $\mathbb{R}_t$  stands for the time direction,  $\mathbb{R}_\phi$  is the linear dilaton theory with central charge  $c = 1 + 3Q^2/2$ , and  $\mathbf{X}$  is a compact SCFT with central charge  $\hat{c} = 3$ . Here and in the rest of the paper we set  $\alpha' = 1$ , hence  $Q^2 = 4/k = 4$ . In (1.3)  $\mathbf{S}^1$  is a boson compactified at the self-dual radius, which is necessary for space-time supersymmetry [6] (for a recent discussion of non-critical superstrings see [7]). To cap the strong coupling region of the linear dilaton theory we introduce the  $\mathcal{N} = 2$  Liouville perturbation in  $\mathbb{R}_\phi \times \mathbf{S}^1$ . Equivalently, we modify the geometry (1.3) to

$$\mathbb{R}_t \times \frac{SL(2)}{U(1)} \times \mathbf{X} \quad (1.4)$$

which asymptotes to (1.3) as  $\phi \rightarrow \infty$ , and set the string coupling at the tip of the cigar to a small but finite value  $g_s$ . The high energy density of perturbative string states in the background (1.4) is given by

$$\rho(M) \sim \frac{e^{\beta_H M}}{M^2} \quad (1.5)$$

with  $\beta_H$  given by (1.2). The appearance of  $M^2$  in the denominator is due to the fact that the linear dilaton direction has a continuous spectrum of excitations above the gap. Various derivations of (1.5) are reviewed in Appendix.

One can consider the euclidean black hole in the background (1.4). When the black hole energy is large,

$$E \gg g_s^{-2} \quad (1.6)$$

the geometry is well described by

$$\frac{SL(2)_{k=1}}{U(1)} \times \mathbf{S}^1 \times \mathbf{X} \quad (1.7)$$

This is because the energy of the black hole is related to the coupling at the tip of the thermal cigar,  $SL(2)_{k=1}/U(1)$ , as  $E \sim \exp(-2\Phi_0)$ , and the effects of the original Liouville wall disappear in the limit (1.6). In other words, the black hole geometry in (1.7) cuts off the linear dilaton direction way before the difference between (1.3) and (1.4) becomes significant. In the next section we review thermodynamics of (1.7), which exhibits Hagedorn density of states

$$\rho_{BH}(E) \sim E^\alpha e^{\beta E} \quad (1.8)$$

with  $\beta$  given by (1.1) and  $\alpha$  being a negative number. In our case  $k = 1$  and hence  $\beta = \beta_H$ .

For energies sufficiently small so that both string self-interaction and non-perturbative effects are negligible, the perturbative closed string states are the correct degrees of freedom, and the density of states is given by (1.5). Once the energy is raised to satisfy (1.6), the black hole picture becomes more appropriate, and the density of states is described by (1.8). Thus it is natural to expect that the entropy interpolates between the expressions (1.5) and (1.8) as the energy is varied.<sup>2</sup> The situation is similar to the higher dimensional case, where the description also interpolates between the perturbative string states and black holes as the energy is increased [2,9]. The main feature of the two-dimensional case that we consider is that the interpolation only happens in the subleading term, since  $\beta$  in (1.8) is equal to  $\beta_H$  in (1.5).

It is interesting to compare  $\alpha$  in (1.8) with  $-2$  which appears in (1.5). In the next Section we consider the simplest case of superstrings in the background (1.7) with  $\mathbf{X} = \mathbf{T}^3$ . Hagedorn density of states (1.8) implies that the free energy  $\mathcal{F}$  of (1.7) is zero at leading order. The first non-vanishing term is

$$\beta\mathcal{F} = -(\alpha + 1) \log E = -Z^{(1)} \quad (1.9)$$

where  $Z^{(1)}$  is the one-loop string partition function in the background (1.7). We compute  $Z^{(1)}$  and determine the value of  $\alpha$ . When the volume of  $\mathbf{T}^3$  is large in string units,  $|\alpha| \gg 1$ . The minimal value of  $|\alpha|$  is achieved when all the compactification radii take the self-dual values. We discuss our results in Section 3. Appendix contains a brief review of various ways of computing and interpreting the string partition function.

## 2. String partition function in the black hole background

In this section we compute the string partition function. The discussion will closely follow [10] where analogous computations were performed for Little String Theory, in the regime  $\beta_H \ll \beta$ . We are interested in the  $\beta_H = \beta$  case, where  $Z^{(1)}$  involves a complicated integral. Although one can correctly estimate the behavior of the integrand, the value of the integral can only be obtained numerically.

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<sup>2</sup> In the BPS case the density of the perturbative states can be matched to that of the black hole beyond the leading order in energy (for a recent discussion see [8]), due to non-renormalization.

The bosonic matter content of the theory is given by (1.7) with  $\mathbf{X} = \mathbf{T}^3$ . The energy of the black hole is related to the coupling at the tip of the cigar as  $E \sim \exp(-2\Phi_0)$ , with the precise coefficient being unimportant. Far from the tip of the cigar, the  $SL(2)_1/U(1)$  is described by a product of a thermal circle and a linear dilaton,  $\mathbf{S}_\beta^1 \times \mathbb{R}_\phi$ . In addition, there are six free fermions; thermal boundary conditions along the  $\mathbf{S}_\beta^1$  are implemented in the usual way [11]. The theory enjoys  $\mathcal{N} = 2$  worldsheet supersymmetry, which ensures vanishing of the genus zero partition function [10]. This is in accord with Hagedorn density of states (1.8): at leading order the temperature is independent of the energy and  $\beta\mathcal{F} = \beta E - S = 0$ . The one-loop string partition function is in general non-zero, and proportional to the volume of the linear dilaton direction (which, in turn, is proportional to  $\log E$ ). The computation, and the resulting thermodynamic behavior, is similar to the Little String Theory case which has been studied in [10,12-20].

The spacetime supersymmetric one-loop partition function at zero temperature can be found in [21]:

$$Z = V_{\mathbb{R}_t \times \mathbb{R}_\phi} \int_{\mathcal{F}_0} \frac{d^2\tau}{2\tau_2} \frac{1}{(4\pi^2\tau_2)} \frac{1}{4|\eta(\tau)|^{12}} (|\Lambda_1(\tau)|^2 + |\Lambda_2(\tau)|^2) \prod_{i=1}^3 \Theta(R_i, \tau) \quad (2.1)$$

where

$$\Lambda_1(\tau) = \Theta_{1,1}(\tau) (\theta_3^2(\tau) + \theta_4^2(\tau)) - \Theta_{0,1}(\tau) \theta_2^2(\tau) \quad (2.2)$$

and  $\Lambda_1(\tau)$  is related to its modular transform under  $\tau \rightarrow -1/\tau$ ,

$$\Lambda_2(\tau) = \Theta_{0,1}(\tau) (\theta_3^2(\tau) - \theta_4^2(\tau)) - \Theta_{1,1}(\tau) \theta_2^2(\tau) \quad (2.3)$$

In (2.1)–(2.3)

$$\Theta_{m,1} = \sum_{n \in \mathbb{Z}} e^{2\pi i \tau (n+m/2)^2} \quad (2.4)$$

and

$$\Theta(R_i, \tau) = \sum_{n,w=-\infty}^{\infty} \exp \left[ -\pi\tau_2 \left( \frac{n^2}{R^2} + w^2 R^2 \right) + 2\pi i \tau_1 n w \right] \quad (2.5)$$

Partition function (2.1) corresponds to the GSO projection by  $(-)^{F+F_{st}}$  where  $F$  and  $F_{st}$  are the worldsheet and spacetime fermion numbers, respectively. There are four massless bosons in both the NSNS and RR sector. Both (2.2) and (2.3) are equal to zero, so that the partition function (2.1) vanishes as expected from spacetime supersymmetry.

Finite temperature results in the extra summation over momentum and winding with respect to the thermal circle. Let us introduce

$$\Lambda_1^{(n,m)}(\tau) = \Theta_{1,1}(\tau) [U_3(n,m)\theta_3^2(\tau) + U_4(n,m)\theta_4^2(\tau)] - \Theta_{0,1}(\tau)U_2(n,m)\theta_2^2(\tau) \quad (2.6)$$

and

$$\Lambda_2^{(n,m)}(\tau) = \Theta_{0,1}(\tau) [U_3(n,m)\theta_3^2(\tau) - U_4(n,m)\theta_4^2(\tau)] - \Theta_{1,1}(\tau)U_2(n,m)\theta_2^2(\tau) \quad (2.7)$$

where  $U_\mu(n,m)$  are the phase factors which can be found in [11]:

$$\begin{aligned} U_1(n,m) &= \frac{1}{2} (-1 + (-1)^n + (-1)^m + (-1)^{n+m}) \\ U_2(n,m) &= \frac{1}{2} (1 - (-1)^n + (-1)^m + (-1)^{n+m}) \\ U_3(n,m) &= \frac{1}{2} (1 + (-1)^n + (-1)^m - (-1)^{n+m}) \\ U_4(n,m) &= \frac{1}{2} (1 + (-1)^n - (-1)^m + (-1)^{n+m}) \end{aligned} \quad (2.8)$$

The free energy (1.9) is given by

$$\beta\mathcal{F} = -\beta L_\phi \int_{\mathcal{F}_0} \frac{d^2\tau}{32\pi^2\tau_2^2|\eta(\tau)|^{12}} \sum_{m,n=-\infty}^{\infty} \left( |\Lambda_1^{(n,m)}(\tau)|^2 + |\Lambda_2^{(n,m)}(\tau)|^2 \right) e^{-S_\beta(n,m)} \prod_{i=1}^3 \Theta(R_i, \tau) \quad (2.9)$$

where the volume of the linear dilaton direction

$$L_\phi = -\frac{1}{Q} \log E + \text{const} \cong -\frac{1}{2} \log E \quad (2.10)$$

and

$$S_\beta(n,m) = -\frac{\beta^2|m - n\tau|^2}{4\pi\tau_2} \quad (2.11)$$

Using modular invariance of the integrand in (2.9), the sum over  $n$  can be restricted to  $n = 0$ , and the integration domain changed to  $-1/2 < \tau_1 < 1/2$ ,  $0 < \tau_2 < \infty$ . The sum over even  $m$  in (2.9) vanishes. Substituting  $\beta = 2\pi$  in (2.9) and comparing with (1.9) we have

$$\alpha + 1 = - \int \frac{d^2\tau}{32\pi\tau_2^2|\eta(\tau)|^{12}} \sum_{m \in 2\mathbb{Z}+1} \left( |\Lambda_1^{(0,1)}(\tau)|^2 + |\Lambda_2^{(0,1)}(\tau)|^2 \right) e^{-\frac{\pi m^2}{\tau_2}} \prod_{i=1}^3 \Theta(R_i, \tau) \quad (2.12)$$

To estimate the behavior of the integrand as  $\tau_2 \rightarrow \infty$  note that only  $n = w = 0$  terms in (2.5) contribute in this regime. The sum over  $m$  in (2.12) needs to be Poisson resummed, giving

an extra factor of  $\sqrt{\tau_2}$ . The terms corresponding to the massive states are exponentially suppressed. To summarize, as  $\tau_2 \rightarrow \infty$  the integrand in (2.12) is dominated by massless states and goes as  $\tau_2^{-3/2}$ . In the case studied in [10] this was the end of story, as  $\beta$  was much larger than  $\beta_H$  and hence the integrand was heavily suppressed near  $\tau_2 = 0$ . It is no longer the case, and the integrand, in fact, behaves like  $\tau_2^{-1/2}$  as  $\tau_2 \rightarrow 0$ . This is in accord with the behavior of the string partition function as  $\beta \rightarrow \beta_H$ , as explained in Appendix. One can also see this directly from (2.12). The naive power counting of  $\tau_2$  gives

$$\tau_2^{-1} \tau_2^{-1} \tau_2^{-4} \tau_2^4 \sim \tau_2^{-2} \quad (2.13)$$

where the first factor comes from the measure, the second from the  $\mathbb{R}_\phi \times \mathbf{S}_\beta^1$ , the third from the  $\mathbf{S} \times \mathbf{T}^3$ , and the last from the combinations of  $\theta_\mu(\tau)$  and  $\eta(\tau)$ . One must be more careful, however, as there is an important exponential

$$\exp\left(\frac{\pi i}{\tau} - \frac{\pi}{\tau_2}\right) \sim \exp\left(-\frac{\pi \tau_1^2}{|\tau|^2 \tau_2}\right) \quad (2.14)$$

which stays constant along the curves  $\tau_1 = x \tau_2^{3/2}$  parameterized by  $x$ . Switching to the variables  $x$  and  $\tau_2$  one earns the Jacobian  $J = \tau_2^{3/2}$ . Multiplying this by (2.13) gives the stated behavior near  $\tau_2 \sim 0$ .

All string states contribute to  $\alpha$  in (2.12) and no further simplifications occur. Generally the right-hand side of (2.12) is proportional to the compactification volume, therefore at large volume  $\alpha$  is bound to be a large negative number. The minimum of  $|\alpha|$  is achieved when all the radii take the self-dual values,  $R_i = 1$ . The integral at this point can be evaluated numerically, giving  $\alpha = -1.68$ .

### 3. Discussion

In this paper we considered a particular case of spacetime-supersymmetric noncritical strings in the background (1.4) whose perturbative Hagedorn temperature is equal to the Hawking temperature of the black hole which asymptotes to (1.4) at infinity. This required choosing the matter content with  $\hat{c} = 4$  to dress the linear dilaton theory with  $Q = 2$ . To cut off the strong coupling region we introduced the  $\mathcal{N} = 2$  Liouville wall in the background (1.3) or, equivalently, considered the geometry (1.4) with small but finite coupling  $g_s$  at the tip of the  $SL(2)/U(1)$  cigar. Massive black holes in this background are described

by the euclidean geometry (1.7). The subleading correction to the black hole entropy is related to the value of the string one-loop partition function in the background (1.7)<sup>3</sup>.

In the black hole/string transition picture of Refs. [1,2,9] the correct degrees of freedom interpolate between the perturbative string states and black holes as the energy is increased. More precisely, the gravitational self-interaction of the string becomes important when  $M > 1/g_s^{\frac{4}{6-d}}$  where  $d$  is the number of non-compact spatial dimensions [2]. In addition, non-perturbative states become important when  $M > 1/g_s$ . In our case  $d = 1$ , so both effects make the perturbative string density of states (1.5) unreliable way before the black hole becomes a good description at  $M \gg 1/g_s^2$ . We expect the density of states to interpolate between the perturbative string expression [Eq. (1.5)] and its black hole counterpart [Eq. (1.8)] as the energy is increased. Since the leading exponential behavior is the same, thanks to  $\beta = \beta_H$ , the interpolation apparently takes place in the subleading term. This should be contrasted with the higher-dimensional case, where the leading behavior of the entropy is different for perturbative strings and black holes.

As a specific model, we considered superstrings in the background (1.4) with  $\mathbf{X} = \mathbf{T}^3$ . The value of  $\alpha$  depends on the volume of compactification. Compactification at the self-dual radii yields  $\alpha = -1.68$ . When the volume of  $\mathbf{T}^3$  becomes large,  $\alpha$  scales like  $-V_{\mathbf{T}^3}$ <sup>4</sup>. Other choices of  $\hat{c} = 4$  matter may lead to other values of  $\alpha$ . In addition to superstrings, we have also considered type 0 and bosonic strings compactified on  $\mathbf{T}^4$  and  $\mathbf{T}^{18}$  respectively, where similar black hole/string transitions occur. These cases, however, are pathological due to the presence of tachyon in the spectrum, which has to be removed by hand.

It is interesting that the numerical value of  $\alpha$  at the self-dual compactification radii computed in the previous Section ( $\alpha = -1.68$ ) is not very different from the corresponding quantity for the perturbative strings ( $\alpha = -2$ ), and therefore even the subleading term in the entropy does not change drastically as the energy is varied.

What happens when  $k > 1$  and the leading terms in the black hole and string entropies no longer agree? As before, we cut off the strong coupling region by introducing a cigar geometry with the string coupling at the tip of the cigar equal to a small but finite value  $g_s$ . The picture is similar to the  $k = 1$  case discussed above. When  $M \gg 1/g_s^2$ , the black

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<sup>3</sup> The perturbative calculation of the black hole entropy is, strictly speaking, only valid for temperatures larger than  $\beta_H^{-1}$ . Nevertheless, the microcanonical description is well defined for both strings and black hole. Using formal inverse Laplace transform to obtain the value of  $\alpha$  might be a point of concern. We have nothing new to say regarding this issue.

<sup>4</sup>  $\alpha = -2$  when  $R_i^2 \approx 3.0$ .



holes are good degrees of freedom and the Hagedorn temperature is now smaller than that of perturbative strings. For sufficiently small energies the perturbative strings become the correct degrees of freedom. We expect the entropy to interpolate between the perturbative string and the black hole expressions.

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## Appendix A. Comments on one-loop string partition function

Standard arguments [22] imply that the number of closed string states at level  $n$  in superstring theory compactified to  $D$  spacetime dimensions is (see e.g. [23])

$$d_n \sim n^{-\frac{1}{2}(D+1)} e^{C\sqrt{n}} \quad (\text{A.1})$$

where  $C$  is an easily computable constant which determines the Hagedorn temperature of perturbative strings in the linear dilaton background (1.2). We are interested in the  $D = 2$  case. Eq. (A.1) translates into the mass density  $\rho(M) \sim M^{-2} e^{\beta_H M}$ . The contribution of high energy states to the free energy of the string gas is

$$\log Z \sim \int dM M^{-2} \exp(\beta_H M) \int_0^\infty dk \exp\left(-\beta \sqrt{k^2 + M^2}\right) \quad (\text{A.2})$$

We are interested in the behavior of (A.2) as  $\beta \rightarrow \beta_H$ :

$$\log Z \sim \int dE E^{-3/2} \exp((\beta_H - \beta)E) \sim (\beta - \beta_H)^{1/2} \quad (\text{A.3})$$

The same answer follows from the thermal scalar calculation [2], where the power of  $(\beta - \beta_H)$  is determined by a number of dimensions with continuous spectra of excitations.

According to Polchinski [24] (A.2) is equal to the one-loop string partition function in the background (1.3). This is what we were computing in Section 2, and indeed the behavior (A.3) is consistent with the  $\tau_2 \rightarrow 0$  asymptotics of the integrand in (2.12):

$$\int d\tau_2 \tau_2^{-1/2} \exp\left(\frac{\beta_H - \beta}{\tau_2}\right) \sim (\beta - \beta_H)^{1/2} \quad (\text{A.4})$$

The fact that (A.3) vanishes as  $\beta \rightarrow \beta_H$  means that all states contribute to the free energy giving a constant which has been omitted in (A.3); that is why we had to resort to numerical methods to evaluate the integral (2.12).

## References

- [1] L. Susskind, Phys. Rev. D **49**, 6606 (1994) [arXiv:hep-th/9308139]; “Some speculations about black hole entropy in string theory,” arXiv:hep-th/9309145.
- [2] G. T. Horowitz and J. Polchinski, Phys. Rev. D **55**, 6189 (1997) [arXiv:hep-th/9612146]; “Self gravitating fundamental strings,” Phys. Rev. D **57**, 2557 (1998) [arXiv:hep-th/9707170].
- [3] D. Kutasov, “Accelerating branes and the string / black hole transition,” arXiv:hep-th/0509170.
- [4] A. Giveon and D. Kutasov, “The charged black hole / string transition,” arXiv:hep-th/0510211.
- [5] A. Giveon, D. Kutasov, E. Rabinovici and A. Sever, “Phases of quantum gravity in AdS(3) and linear dilaton backgrounds,” Nucl. Phys. B **719**, 3 (2005) [arXiv:hep-th/0503121].
- [6] D. Kutasov and N. Seiberg, “Noncritical Superstrings,” Phys. Lett. B **251**, 67 (1990).
- [7] N. Itzhaki, D. Kutasov and N. Seiberg, “Non-supersymmetric deformations of non-critical superstrings,” arXiv:hep-th/0510087.
- [8] H. Ooguri, A. Strominger and C. Vafa, “Black hole attractors and the topological string,” Phys. Rev. D **70**, 106007 (2004) [arXiv:hep-th/0405146].
- [9] T. Damour and G. Veneziano, “Self-gravitating fundamental strings and black holes,” Nucl. Phys. B **568**, 93 (2000) [arXiv:hep-th/9907030].
- [10] D. Kutasov and D. A. Sahakyan, “Comments on the thermodynamics of little string theory,” JHEP **0102**, 021 (2001) [arXiv:hep-th/0012258].
- [11] J. J. Atick and E. Witten, “The Hagedorn Transition And The Number Of Degrees Of Freedom Of String Theory,” Nucl. Phys. B **310**, 291 (1988).
- [12] J. M. Maldacena and A. Strominger, “Semiclassical decay of near-extremal five-branes,” JHEP **9712**, 008 (1997) [arXiv:hep-th/9710014].
- [13] O. Aharony and T. Banks, “Note on the quantum mechanics of M theory,” JHEP **9903**, 016 (1999) [arXiv:hep-th/9812237].
- [14] T. Harmark and N. A. Obers, “Hagedorn behaviour of little string theory from string corrections to NS5-branes,” Phys. Lett. B **485**, 285 (2000) [arXiv:hep-th/0005021].
- [15] M. Berkooz and M. Rozali, “Near Hagedorn dynamics of NS fivebranes, or a new universality class of coiled strings,” JHEP **0005**, 040 (2000) [arXiv:hep-th/0005047].
- [16] M. Rangamani, “Little string thermodynamics,” JHEP **0106**, 042 (2001) [arXiv:hep-th/0104125].
- [17] A. Buchel, “On the thermodynamic instability of LST,” arXiv:hep-th/0107102.
- [18] K. Narayan and M. Rangamani, “Hot little string correlators: A view from supergravity,” JHEP **0108**, 054 (2001) [arXiv:hep-th/0107111].

- [19] P. A. DeBoer and M. Rozali, “Thermal correlators in little string theory,” *Phys. Rev. D* **67**, 086009 (2003) [arXiv:hep-th/0301059].
- [20] O. Aharony, A. Giveon and D. Kutasov, “LSZ in LST,” *Nucl. Phys. B* **691**, 3 (2004) [arXiv:hep-th/0404016].
- [21] S. Mizoguchi, “Modular invariant critical superstrings on four-dimensional Minkowski space x two-dimensional black hole,” *JHEP* **0004**, 014 (2000) [arXiv:hep-th/0003053].
- [22] M. B. Green, J. H. Schwarz and E. Witten, “Superstring Theory. Vol. 1: Introduction,”
- [23] K. R. Dienes, E. Dudas, T. Gherghetta and A. Riotto, “Cosmological phase transitions and radius stabilization in higher dimensions,” *Nucl. Phys. B* **543**, 387 (1999) [arXiv:hep-ph/9809406].
- [24] J. Polchinski, “Evaluation Of The One Loop String Path Integral,” *Commun. Math. Phys.* **104**, 37 (1986).